# Topological vector spaces and applications to geometry

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The traditional functional analysis deals mostly with Banach spaces and, in particular, Hilbert spaces. However, many classical vector spaces have canonical topologies that cannot be determined by a single norm. For example, many spaces of smooth functions, holomorphic functions, and distributions belong to the above class. Such spaces are the subject of the theory of topological vector spaces. Although the golden age of topological vector spaces was in the 1950ies, their theory is still evolving nowadays, contrary to a stereotyped view coming from incompetent sources. The current development of topological vector spaces is directed not so much towards general theory as towards applications in PDEs and in complex analytic geometry.

We plan to discuss the basics of the theory of topological vector spaces, including some applications to complex analytic geometry.

**Prerequisites.** Basic functional analysis (Banach and Hilbert spaces, bounded linear operators).

## Syllabus

1. **Basic notions and examples.** Topological vector spaces. Seminorms and locally convex spaces. Continuous linear maps. Normability and metrizability criteria. Completeness. Examples: spaces of continuous, smooth, holomorphic functions, the Schwartz space.

2. **Constructions.** Quotients, products, coproducts, inverse and direct limits, completions, topological tensor products.

3. **Linear maps.** Bornological and barrelled spaces. Equicontinuity. The Banach-Steinhaus theorem, the Open Mapping theorem, the Banach-Alaoglu-Bourbaki theorem.

4. **Duality.** Dual pairs and weak topologies. The Bipolar theorem. The Mackey-Arens theorem. The Mackey topology, the strong topology. Reflexivity. Relations between properties of linear maps and their duals.

5. **Tensor products and nuclear spaces.** Nuclear maps. Nuclear spaces and their properties. Examples of nuclear spaces. A characterization of nuclear spaces in terms of tensor products.

6. **Applications.** Coherent analytic sheaves. The Cartan-Serre finiteness theorem. The Grauert direct image theorem[1].

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[1] if time permits