C^*-ALGEBRAS AND COMPACT QUANTUM GROUPS

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C^*-algebras

[Murphy] G. J. Murphy. C^*-algebras and operator theory. Academic Press, 1990. Russian transl.: Factorial, 1997. *One of the best elementary introductions to C^*-algebras.*

[Davidson] K. R. Davidson. C^*-algebras by example. AMS, 1996. *Less elementary than Murphy's book. After a brief introduction to the basics of C^*-algebras, the author proceeds to a thorough discussion of numerous examples and to more advanced topics.*

[Blackadar] B. Blackadar. Operator algebras. Springer, 2006. *The book is written in the style of the famous Russian series "Itogi nauki i tehniki, Sovremennye problemy matematiki, Fundamental'nye napravleniya" (some volumes of the series were translated into English under the title "Encyclopaedia of Mathematical Sciences"). In other words, this is a mixture of a monograph, a survey, and a reference book. Most proofs are either omitted or sketched. This enabled the author to cover a variety of diverse topics in a reasonably-sized volume.*

[Dixmier] J. Dixmier. C^*-algebras and their representations. North-Holland, 1977. Russian transl.: Nauka, 1974. *One of the first modern introductions to the C^*-algebra theory (the 1st French edition appeared in 1964).*

[KadRingr] R. V. Kadison and J. R. Ringrose. Fundamentals of the theory of operator algebras. Academic Press, 1983 (Vol. 1), 1986 (Vol. 2). *A "bible" of operator algebras. Contains many exercises. The only strange feature of the book is that all C^*-algebras are assumed to be unital.*

[Takesaki] M. Takesaki. Theory of operator algebras. Springer, 2002 (vol. I), 2003 (vols. II and III). *Another comprehensive monograph on operator algebras. Less elementary than Kadison and Ringrose's book, and covers much more advanced topics.*

[Helemskii] A. Ya. Helemskii. Banach and locally convex algebras. Oxford, 1993. Russian original: Nauka, 1989. *C^*-algebras are treated from the viewpoint of the general theory of Banach $*$-algebras.*

[FD] J. M. G. Fell, R. S. Doran. Representations of $*$-algebras, locally compact groups, and Banach $*$-algebraic bundles. Academic Press, 1988. *A two-volume monograph on representations of various functional-analytic structures. Chapter VI is devoted to C^*-algebras.*

[Lance] E. C. Lance. Hilbert C^*-modules. A toolkit for operator algebraists. Cambridge Univ. Press, 1995. *A short and user-friendly introduction to Hilbert C^*-modules, with a view towards quantum groups.*

Compact quantum groups

[Timmermann] T. Timmermann. An invitation to quantum groups and duality. EMS, 2008. *The first book entirely devoted to the functional-analytic aspect of (compact and locally compact) quantum groups.*

[NeshvTuset] S. Neshveyev, L. Tuset. Compact quantum groups and their representation categories. SMF, 2013. *Compared to Timmermann's book, this is a more detailed introduction to compact quantum groups. Noncompact quantum groups are not mentioned. On the other hand, the book contains a full treatment of the Tannaka-Krein duality for compact quantum groups.*

[KlimSchmdgn] A. Klimyk, K. Schmüdgen. Quantum groups and their representations. Springer, 1997. *A fundamental monograph on representations of quantum groups. The emphasis is on the algebraic aspects, but compact quantum groups in Woronowicz's sense are also discussed.*

[MVD] A. Maes, A. Van Daele. Notes on compact quantum groups. Nieuw Arch. Wisk. (4) 16 (1998), no. 1–2, 73–112. *An expository paper on compact quantum groups. More general locally compact qunatum groups are also briefly mentioned.*

[WorSU2] S. L. Woronowicz. Twisted SU(2) group. An example of a non-commutative differential calculus. Publ. RIMS, Kyoto Univ. **23** (1987), 117–181. *Here the quantum* SU(2) *was introduced for the first time.*

[WorMatr] S. L. Woronowicz. Compact matrix pseudogroups. Commun. Math. Phys. **111** (1987), 613-665. *A general theory of compact matrix quantum groups is developed.*

[WorTann] S. L. Woronowicz. Tannaka-Krein duality for compact matrix pseudogroups. Twisted SU(N) groups. Invent. Math. **93** (1988), 35–76. *The title is self-explanatory.*

[WorCQG] S. L. Woronowicz. Compact quantum groups. In: "Symétries quantiques" (Les Houches, 1995), 845–884, North-Holland, 1998. *The general notion of a compact quantum group is introduced, and a relation with the earlier matrix approach is established.*